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SIXTH ANNUAL MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA.

By invitation of the University of Toronto and the Royal Canadian Institute, the sixth annual meeting of the Mathematical Association of America was held at the University of Toronto on Thursday and Friday, December 29 and 30, 1921, in affiliation with the American Association for the Advancement of Science and in connection with the annual meeting of the American Mathematical Society. There were 110 in attendance at the sessions, including the following 88 members of the Association:

R. C. Archibald, Brown University.
G. N. Armstrong, Ohio Wesleyan University.
C. S. Atchison, Washington and Jefferson College.
Clara L. Bacon, Goucher College.
L. A. Bauer, Dept. of Terrestrial Magnetism, Washington, D. C.
Samuel Beatty, University of Toronto.
R. D. Beetle, Dartmouth College.
O. F. H. Bert, Washington and Jefferson College.
G. A. Bliss, University of Chicago.
J. W. Bradshaw, University of Michigan.
R. W. Burgess, Brown University.
W. D. Cairns, Oberlin College.
H. C. Carver, University of Michigan.
Louise D. Cummings, Vassar College.
C. H. Currier, Brown University.
D. R. Curtiss, Northwestern University.
A. T. De Lury, University of Toronto.
W. W. Denton, University of Michigan.
Sister Mariola Dobbin, St. Clara College.
B. F. Dostal, University of Denver.
L. H. Dube, Ottawa University.
Arnold Dresden, University of Wisconsin.
Peter Field, University of Michigan.
J. C. Fields, University of Toronto.
William Findlay, McMaster University.
B. F. Finkel, Drury College.
T. M. Focke, Case School of Applied Science.
W. B. Ford, University of Michigan.
J. L. Gibson, University of Utah.
D. C. Gillespie, Cornell University.
O. E. Glenn, University of Pennsylvania.
J. W. Glover, University of Michigan.
R. A. Gray, Oakwood Collegiate Institute, Toronto.
C. F. Gummer, Queen's University.
Olive C. Hazlett, Mt. Holyoke College.
E. R. Hedrick, University of Missouri.
H. A. Howe, University of Denver.
E. V. Huntington, Harvard University.
W. A. Hurwitz, Cornell University.
M. H. Ingraham, University of Wisconsin.
G. H. Jamison, Kirksville (Mo.) State Teachers College.
L. C. Karpinski, University of Michigan.
G. W. Keith, Parkdale Collegiate Institute, Toronto.
J. P. Kelly, Boston College.
Jennie A. Kinnear, Port Colborne, Ont.
H. R. Kingston, Western University.
N. J. Lennes, University of Montana.
Florence P. Lewis, Goucher College.
E. J. Maurus, Notre Dame University.
G. A. Miller, University of Illinois.
Norman Miller, Queen's University.
G. R. Mirick, East High School, Rochester, N. Y.
E. H. Moore, University of Chicago.
F. R. Moulton, University of Chicago.
B. L. Newkirk, General Electric Co., Schenectady, N. Y.
H. L. Olson, University of Michigan.
F. W. Owens, Cornell University.
Mrs. F. W. Owens, Cornell University.
W. J. Patterson, Western University.
Mrs. Anna J. Pell, Bryn Mawr College.
H. B. Phillips, Mass. Institute of Technology.
A. D. Pitcher, Western Reserve University.
I. R. Pounder, University of Toronto.
P. L. Rea, Marietta College.
A. V. Richardson, Bishop's College.
R. G. D. Richardson, Brown University.
H. L. Rietz, University of Iowa.
G. M. Robison, Cornell University.
E. D. Roe, Jr., Syracuse University.
W. H. Roever, Washington University.
N. E. Sheppard, University of Toronto.
Mary E. Sinclair, Oberlin College.
H. E. Slaught, University of Chicago.
W. A. Skirrow, Parkdale Collegiate Institute, Toronto.
Clara E. Smith, Wellesley College.
E. R. Smith, State College of Iowa.
I. W. Smith, North Dakota Agric. College.

P. F. Smith, Yale University.
H. S. Uhler, Yale University.
Oswald Veblen, Princeton University.
G. L. Wagar, Mt. Hermon School.
C. W. Watkeys, Rochester University.
A. G. Webster, Clark University.
E. T. White, London Normal School.
C. E. Wilder, Northwestern University.
A. H. Wilson, Haverford College.
E. B. Wilson, Mass. Institute of Technology.
Jessica M. Young, Washington University.

The meetings of the American Association were held during the week, beginning with the address of the Retiring President on Tuesday evening, and continuing through Saturday of that week. There was a total attendance of over 1800 at the various sessions of the American Association and affiliated societies. The University authorities and the Royal Canadian Institute made exceptionally fine preparations for the entertainment of visitors, and an attractive combination of scientific programs and social diversions was furnished for the various groups of scientists. On Wednesday evening in Convocation Hall, Professor William Bateson, director of the John Innes Horticultural Institute of Surrey, England, delivered an address on "Evolutionary Faith and Modern Doubts." This address appears in full in *Science* for January 20, 1922. At the close of this address a Convocation of the University of Toronto was held, presided over by Sir Robert Falconer, at which the degree of Doctor of Science *honoris causa* was conferred on Professor Bateson, Retiring President A. A. Howard, and President E. H. Moore. On Wednesday afternoon at the close of the mathematical program, a reception was given in Hart House at which the mathematicians were cordially received by Sir Robert and Lady Falconer, and Professor A. T. De Lury. At a general session on Thursday afternoon, Sir Adam Beck, Chairman of the Hydro-Electric Commission of Ontario, gave an address on "Hydro-Electric Developments in Ontario," illustrating by a series of moving pictures the various hydro-electric projects in Ontario. The largest of the social functions was the Thursday evening conversazione in Hart House where the many guests of the University and the Royal Canadian Institute were entertained in a variety of fascinating ways in the large halls and theater of Hart House. At the close of the mathematical program on Friday evening, an exhibition of fancy skating and a hockey match between the Varsity Intermediates and the St. Helens Intermediates was held at the Arena. All arrangements for the convenience of guests were ably made by the Committee on Arrangements, under the chairmanship of Professor De Lury. A special committee of ladies of the University paid constant attention to the entertainment of the ladies among the visiting guests, and a general dinner for the ladies was given Friday evening at the same time as the dinners of the various groups of scientists. The mathematicians were housed in Knox College and in Annesley Hall on the Campus, and enjoyed thus the same social conveniences that have characterized a number of the summer meetings of the Association and Society. The joint dinner of mathematicians, astronomers, and physicists on Friday evening was attended by about 150 guests. During the dinner the visitors were entertained by the Adanac quartet, Miss Vera McLean, and Mrs. Collinshaw, of Toronto. At the closing session of the Association a resolution of thanks to the mathematical staff and the authorities of the University of Toronto, to the Royal Canadian Institute, and to the Program Committee, was unanimously adopted. A very extensive account of the features of the Toronto meeting of the American Association is given in *Science* for January 13 and 20, 1922.

Many members attended the joint symposium on Quantum Theory on Thursday morning, at which Section C of the American Association was represented by Professor R. C. Tolman, the American Physical Society by Dr. Saul Dushman, and the American Mathematical Society by Professor H. B. Phillips.

The program was formulated by the Program Committee under the chairmanship of Professor C. F. Gummer of Queen's University. An innovation that was generally commended was the institution of a series of papers giving outlines of certain fields of research. It is frequently urged that College and University teachers should be engaged in some form of productive work, but many college instructors do not know promising lines of investigation, and do not know how and where to find the literature which will inform them of what has already been done in various lines. A suggestion has been made that the Association can do a valuable service if on its programs and through the MONTHLY, university teachers map out for college teachers possible lines of research growing readily out of college courses. The papers by Professors Veblen and Bliss afford a beginning of such suggestions.

The following papers were given. Abstracts of these papers follow, the numbers corresponding to the numbers in the lists of titles.

JOINT SESSION OF THE ASSOCIATION WITH THE AMERICAN MATHEMATICAL SOCIETY AND SECTION A OF THE AMERICAN ASSOCIATION.

(1) "Algebraic guides to transcendental problems," retiring address by Professor R. D. CARMICHAEL, University of Illinois, chairman of the Chicago Section of the Society, and a vice-president of the Association.

(2) "A mechanical analogy in the theory of equations" by Professor D. R. CURTISS, Northwestern University, retiring vice-president of Section A.

(3) "Subsidy funds for mathematical projects" by Professor H. E. SLAUGHT, University of Chicago.

(4) "Informational service of the National Research Council" by Dr. R. M. YERKES, Chairman Research Information Service.

1. Professor Carmichael stated that algebraic guides in transcendental equations had been hitherto used in a very limited number of fields, notably in that of integral equations where the method was used by Volterra and Fredholm as a heuristic guide, but by Hilbert as a method of deducing the results in integral equations by limiting processes from algebraic processes as the number of variables becomes infinite. He instanced earlier uses of analogous methods by Cauchy which were clarified by Lipschitz, by Sturm who passed from the solutions of difference equations to infinitely small differences, from the treatment of linear difference equations to that of differential equations, by Porter who, in the *Annals of Mathematics* in 1902, carried out some of the essential work done more than

two years later by Hilbert. Professor Carmichael discussed two types of problems, in the one class oscillation and comparison theorems, in the other theorems of expansion in orthogonal functions and their generalizations, and in successive sections he formulated algebraic and transcendental oscillation, comparison, and expansion theorems. His paper has been offered to the *Bulletin of the American Mathematical Society.*

2. Professor Curtiss's paper appears in *Science* for February 24, 1922, pp. 189–194.

3. Professor Slaught displayed in graphic manner some ways in which mathematics needs funds for carrying out its projects, by exhibiting the following table and commenting upon it item by item. The paper was printed in full in *Science* for February 10, 1922, pp. 146–148.

SUBSIDY FUNDS NEEDED FOR MATHEMATICAL PROJECTS.

	Annual	Lump Sum	Endowment
1. Revolving book fund for the publication of treatises		$ 25,000	
2. Mathematical Dictionary, already carefully planned by the Association		100,000	
3. Publication of Historical Journal—Continuation of *Bibliotheca Mathematica*	$ 2,000		$ 40,000
4. Enlargement of all the research journals to full capacity	2,000		40,000
Extra volume of the *Transactions* of the American Mathematical Society		4,000	
5. Two extra numbers of the AMERICAN MATHEMATICAL MONTHLY for expository papers and expansion of the regular volume by 80 pages	2,000		40,000
6. Publication of Mathematical Monographs—Funds already provided by Mrs. Mary Hegeler Carus	1,200		24,000
7. *Mathematical Abstract Journal,* approved by the National Research Council	15,000		300,000
8. Preparation of a Bibliography of Bibliographies in Mathematics		5,000	
9. For prizes and Research Fellowships	25,000		500,000
10. Honorary Stipends for Executive Officers of the Society and Association	4,000		80,000
Totals	$51,200	$134,000	$1,024,000

4. Dr. Yerkes stated that the Informational Service of the National Research Council early decided to have not a storehouse but a clearing house of information. They are attempting to develop a personnel file of research activities and achievements of all who have been trained to research, or have attained to research ability, news of centers of interest, progress of current work, information as to the laboratories and institutions which are equipped for various lines of work; and they purpose to keep this information classified under various useful headings. They are seeking to coördinate the existing facilities in our libraries and to make these available through various source-books, and through lists of published and unpublished bibliographies, and thus to avoid the lamentable loss resulting from the constant duplication of effort in finding what has been done in any

particular line. The Service is trying to create a special division to take care of needs as to scientific apparatus. But the principal part of the machinery is the corps of specialists with particular knowledge of the subject and with the insight necessary for interpreting or clarifying the needs of those applying for aid. Mathematicians like other scientists are urgently invited to use the Service for whatever purposes they may wish of a scientific sort.

Session of the Association.

(5) Outlines of Fields of Research: (*a*) "Foundations of geometry" by Professor Oswald Veblen, Princeton University.

(6) Outlines of Fields of Research: (*b*) "Calculus of variations" by Professor G. A. Bliss, University of Chicago.

(7) "Courses in mechanics for students majoring in mathematics" by Professor E. V. Huntington, Harvard University.

(8) "Topology of three-dimensional manifolds in three dimensions" by Professor Norman Miller, Queen's University.

5. Professor Veblen's paper was devoted to a discussion of the field for research in the foundation of elementary geometry. He first referred to the study of the axioms of order by Pasch, Peano, E. H. Moore, and others, and called attention to a set of axioms of betweenness given by C. Müntz (*Jahresbericht der Deutschen Math. Ver.*, 1914) which seem very satisfactory. Aside from further improvements of these axioms (in the direction of "complete independence" for example) there is much to be done in the way of demonstrating the fundamental topological theorems. Among these are to be counted not only problems involving refined continuity considerations such as have been studied by R. L. Moore, N. J. Lennes, J. R. Kline and others, but such combinatorial questions as the classification of all polygons in space. A solution of this problem would be equivalent to a solution of the well-known knot problem of analysis situs. There is thus a whole series of problems, some easy and some very difficult, for anyone who will start with the axioms of order and try to build up the theorems which should immediately follow.

In the researches referred to above there is little if any apparent relation to the problem of teaching elementary geometry. The time has now arrived, however, when a study of the foundations of geometry should be made with the object of obtaining a logical formulation of the subject which can be used in elementary teaching. This is a problem of research because no suitable formulation of the subject yet exists, and it is one in which the routine experience of college and secondary school teachers can be directly useful. The sort of axioms to look for would be less like the order axioms of Pasch than like those given at the beginning of Veblen and Young's *Projective Geometry*. Among the general principles which should be followed in such a treatment of geometry are:

(*a*) The introduction of coördinates and the methods of analytical geometry should come very early. Further assumptions and definitions could then be stated in analytical terms, and the development of algebra and geometry go hand in hand.

(*b*) It should be kept continually in mind that geometry is a branch of natural science. The abstract logical structure should be compared carefully at each stage with the facts of nature. This comparison is in general possible only with a certain percentage of accuracy. It is not enough to verify the axioms and then feel sure that all theorems are true, for it might easily happen that all the axioms are verified within one one hundredth of a per cent. and yet that some of the theorems fail by several per cent. Every important theorem should be tested by experiment or experience just as in physics and in chemistry.

(*c*) A thoroughly skeptical attitude should be preserved toward the facts of geometry. The parts of space about which we speak with confidence are here and now. As soon as we speak of distant parts of space or of space as it was in the distant past we encounter difficulties with regard to the evidence. Those who teach geometry, if not those who are learning it, should be conscious that it gives only a partial description of nature and that this description may have to undergo serious modifications before it can be made a part of a more adequate description.

6. According to Professor Bliss research is the discovery of a problem on which to work rather than the working out of an assigned topic. One needs enthusiasm for such a task, a moderate acquaintance with the literature, and a good stock of imagination. In speaking of the calculus of variations, which was his particular chosen topic, he stated that the earlier stage was that of problem solving, or finding problems like those already solved; the present stage is one in which the general methods of the theory have been or are being attacked, together with the problem of fitting the general theory to special cases, etc. For example: the solution of Newton's solid of revolution of minimum resistance does not provide for the case where the curve is to begin at the x-axis; Bolza found by a not very difficult analysis the solution for this special case.

Professor Bliss gave samples of problems in the calculus of variations which are partly solved, but which still afford opportunity for original work. Four problems of this kind are the shortest line, the brachistochrone (curve of quickest descent), minimum surface of revolution, minimum area between a curve, its evolute and the bounding normals. As an outgrowth from this last problem, Rider and Dunkel have treated the minimum area between a curve and its caustic. Under the other problems, Mason has treated the problem of passing a wire through two points so that the moment of inertia about the origin is a minimum; and under the old problem of minimum surfaces of revolution, MacNeish and Miss Sinclair made extensions of various sorts. Professor Bliss closed with the statement that one can still find examples under the old problems which admit of new extensions, if only one reads up the subject of the calculus of variations and investigates what has been done on these old problems.

Professor Hurwitz corroborated what the two speakers said in reference to the beginnings of research, instancing various problems and discussions which have appeared in recent numbers of the MONTHLY.

Professor Glover stated that the *Transactions of the Actuarial Society of*

America has published a list of about fifty different subjects for research, a list which was obtained from the members of that society, and suggested that a similar procedure might readily produce valuable results for the Association.

7. After pointing out the futility of trying to teach kinetics without the use of the calculus, Professor Huntington expressed the belief that any course in theoretical mechanics ought to aim: (*a*) to provide the student with an opportunity to extend and solidify his knowledge of mathematical methods, especially the calculus; and (*b*) to teach him the practical principles of dynamics as such, unencumbered by technical mathematical details.

Attempts sometimes made to separate these two aims are in danger of leading (in the hands of "pure mathematicians") to a loss of all mechanical perspective in a maze of mathematical manipulation; or else (in the hands of "practical engineers") to a loss of all scientific generality in a multiplicity of numerical cases treated more or less by rule of thumb.

Since the theory of the subject is extremely brief, a course in mechanics is made or marred by its choice of problems. The whole question turns on the selection of good problems—problems which reveal the power and scope of mathematical methods without degenerating into mere mathematical puzzles—problems which yield interesting mechanical results, without becoming merely a series of detached numerical cases. The main part of the paper consisted in a detailed discussion of several examples of typical problems of this sort.

Professor Webster said that mathematicians on going out from college should know the leading problems of historical interest in mechanics, not for the purpose of helping one's calculus, but in order to know what the most important problems of mechanics are, and to know the leading methods of mechanics, whether elementary or advanced. He therefore did not agree with Professor Huntington in the advisability of using such fanciful problems as that of the squirrel and the cage.

8. In the search for conditions for the equivalence of two manifolds in the sense of analysis situs various invariants have been defined. Dr. Miller considered the case of regions of three-dimensional space and showed how certain invariants present themselves naturally in their physical construction. Any region may be built up, in the manner of a clay model, from pieces each equivalent to a sphere. At any stage in the construction an added piece may touch the region already formed along one or more faces, each of which may be bounded by one or more curves. The operations are, however, reducible to three kinds, viz.: (*a*) adding a piece which touches along a simply connected face; (*b*) adding a piece which touches along two simply connected faces, "putting a handle on a surface;" (*c*) adding a piece which touches along a doubly connected face. A region constructed by operations (*a*) alone is simply connected. A region constructed by operations (*a*) and (*b*) only is characterized topologically by one invariant, its connectivity, which may be defined as one more than the number of (*b*) operations used. A region with a single boundary which requires operations (*c*) is not equivalent to a region of the former type. An example of such a region

is the interior of a sphere from which a knotted tube has been removed. The least number of (c) operations which must be used in the construction of a region may be called the degree of knottedness of the region. In order that two regions be equivalent topologically this number must be the same for both. The definition of connectivity is now extended to apply to regions of this second type and the two invariants are defined for regions with several boundaries.

SESSION OF THE ASSOCIATION.

(9) "Functionality in mathematical instruction in schools and colleges" by Professor E. R. HEDRICK, University of Missouri.

(10) "An example in the inversion of upper limits and bounds" by Professor SAMUEL BEATTY, University of Toronto.

(11) "New mathematical periodicals" by Professor G. A. MILLER, University of Illinois.

(12) "Proof of the fundamental theorem regarding the length of a curve" by Professor J. L. SYNGE, University of Toronto, by invitation.

9. In this paper, Professor Hedrick explained and expounded in some detail the principle announced by the National Committee on Mathematical Requirements concerning the central position in all mathematical teaching that is occupied by the function concept. He described the contents of the report of the same committee on The Function Concept in Secondary School Mathematics, which was prepared for the committee by the speaker and was published by the U. S. Bureau of Education as *Secondary School Circular No. 8*, June, 1921. He then proceeded to extend these ideas to higher secondary courses and to collegiate courses in mathematics. He pointed out by the citation of many instances the need for the formation of accurate habits of quantitative thinking and of functional thinking on the part of the great masses of our population, not only for the control of situations that arise in their private lives, but also for efficient judgment of such great public questions as the control of public service corporations, life insurance, railroad rates, pensions, and the like, all of which necessarily involve quantitative relationships that will be misunderstood by voters and legislators unless they have formed correct habits of functional thinking. Finally, the necessary reform of mathematical teaching to insure such training was discussed. Colleges and Universities, as well as secondary schools, must fundamentally alter their traditional mathematical courses if the advantages of mathematical study pointed out in this paper are to be realized.

10. Professor Beatty showed that if $f_1(t), f_2(t), \cdots$ is a sequence of real functions defined for a continuous, closed interval (t), and if $\operatorname*{limit}_{n} \operatorname*{bound}_{(t)} f_n(t)$ is finite, then $\operatorname*{bound}_{(t)} \operatorname*{limit}_{n} f_n(t)$ is equal to or less than it; moreover, a sufficient condition for equality is that, corresponding to each pair t, $\epsilon > 0$, there exists a pair n, $\delta > 0$ such that

$$f_{n+p}(t + \theta\delta) - \overline{\lim_{n}}\, f_n(t) < \epsilon,$$

for every positive integer p and every fraction θ between 1 and -1. This finds application in the general theory of the complex power-series. For suppose that $a_0, a_1, a_2, \cdots$ is a sequence for which

$$\overline{\lim_{n}}\, |a_n{}^{1/n}| = 1,$$

and that A_n of the sequence $A_0, A_1, A_2, \cdots$ denotes

$$a_n + \binom{n+1}{1} a_{n+1}\sigma e^{it} + \binom{n+2}{2} a_{n+2}\sigma^2 e^{2it} + \cdots,$$

σ being an assigned proper fraction and t ranging over the interval $(0, 2\pi)$; then it appears that

$$\overline{\lim_{n}}\; \overline{\underset{(t)}{\text{bound}}}\, |A_n{}^{1/n}| = \overline{\underset{(t)}{\text{bound}}}\; \overline{\lim_{n}}\, |A_n{}^{1/n}|.$$

This leads to a demonstration of the existence of a singularity on the circle of convergence of the complex power-series

$$a_0 + a_1 z + a_2 z^2 + \cdots.$$

11. Professor Miller said that various mathematical journals have been disappearing, others are in great difficulties and are in danger of dying off. While a number of mathematical journals have lived to a ripe and useful old age, the average journal has been short-lived. Among the recently inaugurated journals which are of promising character and are contributed to by many noted and able writers are:

Zeitschrift für Mathematik; Zeitschrift für Angewandte Mathematik und Mechanik; Stanford University Publications (Series in Mathematics and Astronomy) begun in January, 1921; *The Mathematics Teacher,* conducted under the auspices of the National Council of Teachers of Mathematics; the Italian journal *Periodico,* which began a new series this present year of 1921; a Dutch periodical which is to appear six times a year; and others.

An inquiry from Professor Curtiss brought out the fact that the *Archiv* and the *Bibliotheca Mathematica* have apparently been discontinued, but that the *Jahrbuch* is expected to continue. It is, of course, obvious that the status of some European journals is uncertain, and that there is slight information of a definite sort concerning some of the periodicals which have been for many years familiar to American mathematicians.

12. Professor Synge gave an alternative proof of the existence theorem that if a curve is given in the parametric form

$$x = x(u), \qquad y = y(u), \qquad z = z(u),$$

where $\dot{x}$, $\dot{y}$, and $\dot{z}$ exist and are continuous, not all vanishing simultaneously, then the length of the curve from $A(u=u_A)$ to $B\ (u=u_B)$ exists and is given by

$$\int_{u_A}^{u_B} \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, du.$$

MEETING OF THE BOARD OF TRUSTEES OF THE ASSOCIATION.

Ten members of the Board were present at the meetings.

The following 58 persons and 4 institutions, on application duly certified, were elected to membership:

To individual membership.

FRANK AYRES, Jr., B.S. (Washington Coll.). Head of dept. of math., Ogden College, Bowling Green, Ky.

W. E. BAGLEY, M.S. (Washington). Head of dept. of science, Central Academy and College, McPherson, Kans.

R. W. BARNARD, M.S. (Michigan). Instr., Univ. of Michigan, Ann Arbor, Mich.

V. E. BEAUPRÉ, B.A. (Manitoba). Prof., Ecole Polytechnique, Montreal, Canada.

RACHEL BLODGETT, Ph.D. (Radcliffe). Instr., Wellesley College, Wellesley, Mass.

J. E. BURNAM, A.M. (Texas). Asso. prof., Simmons College, Abilene, Tex.

MAY B. CARTER, A.M. (Brown). Asso. prof., The Western College for Women, Oxford, Ohio.

H. C. CARVER, B.S. Prof., math. and insurance, Univ. of Michigan, Ann Arbor, Mich.

JUNG LU CHIN, graduate of Peking Teachers College. Graduate stud., Univ. of Chicago.

W. A. COIT, Ph.D. (Boston). Prof., Acadia Univ., Wolfville, N. S.

J. P. DALTON, M.A., D.Sc. (St. Andrews); D.Sc. (Cape). Prof., Univ. of the Witwatersrand, Johannesburg, S. A.

C. H. DAVIS, American Railway Express Co., Providence, R. I.

A. T. DE LURY, M.A. (Toronto). Prof., University of Toronto, Toronto, Canada.

LEILA EASON, A.B. (Texas). Palestine, Tex.

J. H. EICHELBERGER, Student, Univ. of Chicago. Freeport, Ill.

IVA ERNSBERGER, A.M. (Nebraska). Instr., Non Collegiate, Iowa State College, Ames, Ia.

DAVID FORSYTH, B.A. (Toronto). Principal Emeritus, Kitchener and Waterloo Collegiate Inst., Kitchener, Ont.

R. M. FRÉCHET, Professeur d'Analyse Supérieure, Faculté des Sciences de Strasbourg, Strasbourg, France.

MARGARET L. FURREY, B.S. (Northwestern). Asst. instr., Northwestern Univ., Evanston, Ill.

R. A. GRAY, B.A. (Toronto). Principal, Oakwood Collegiate Inst., Toronto, Ont.

E. S. HAMMOND, Ph.D. (Princeton). Asst. prof., Bowdoin College, Brunswick, Me.

PAULA F. HENRY, A.M. (Texas). Teacher, Fort Worth High School, Fort Worth, Tex.

H. C. Hicks, Ph.B. (Chicago). Grad. student, Univ. of Chicago, Chicago, Ill.

F. C. Householder, A.B. (Kansas). Asso. prof., N. Dak. Agricultural Coll., Fargo, N. Dak.

Marie M. Johnson, M.S. (Iowa). Instr., Lake Forest Coll., Lake Forest, Ill.

G. W. Keith, B.A. (Toronto). Math. master, Parkdale Colleg. Inst., Toronto, Ont.

J. T. Kendrigan, B.S. (Maine). Prof., Northland Coll., Ashland, Wis.

Jennie A. Kinnear, B.A. (Queen's Univ.). Port Colborne, Ont.

Louise Lange, Ph.D. (Göttingen). Prof., Oxford College, Oxford, Ohio.

Arthur Léveillé, B.A. (London). Prof., Univ. of Montreal, Montreal, Canada.

M. A. Mackenzie, M.A. (Cantab.). Prof., Univ. of Toronto, Toronto, Ont.

Max Morris, A.M. (Harvard). Instr., Case School of Applied Science, Cleveland, Ohio.

D. S. Morse, A.M. (Harvard). Instr., Cornell Univ., Ithaca, N. Y.

Thirza A. Mossman, A.B. (Nebraska). Instr., St. Xavier Coll. for Women, Chicago, Ill.

W. J. Patterson, M.A. (Queen's). Prof., Western Univ., London, Canada.

Edna P. Pepper, A.B. (Western Coll.). Instr., Western Coll., Oxford, Ohio.

H. A. Robinson, Senior, Univ. of Ga., Athens, Ga.

Kathleen M. Rose, B.A. (Queen's Univ.). Teacher, Sudbury High School, Sudbury, Ont.

S. M. Sewell, M.S. (Chicago). Prof., Southwest Texas State Normal Coll., San Marcos, Tex.

Leonard Shaffer, A.B., B.S. (Penna.). Instr., New Hampshire Coll., Durham, N. H.

H. C. Shaub, A.B. (Ohio State). Instr., Dartmouth Coll., Hanover, N. H.

R. A. Sheets, A.M. (Denison). Asst. prof., Denison Univ., Granville, Ohio.

N. E. Sheppard, M.A. (Toronto). Lecturer, Univ. of Toronto, Toronto, Ont.

W. A. Skirrow, M.A. (Queen's). Asst. master, Parkdale Colleg. Inst., Toronto, Ont.

G. R. Smith, B.A. (Toronto). Ottawa, Ont.

G. W. Spenceley, A.M. (Harvard). Asst. prof., Miami Univ., Oxford, Ohio.

C. W. Strom, A.B. (Luther Coll.). Instr., Luther Coll., Decorah, Ia.

Margaret F. Temple, A.B. (Hamline). Teacher, High School, Jackson, Minn.

R. J. W. Templin, A.M. (Bucknell). Instr., Lafayette Coll., Easton, Pa.

W. N. Thompson, B.S. (Drury); A.B. (Northwestern). Asst. prof., Drury Coll., Springfield, Mo.

Althéod Tremblay. Prof., Laval Univ., Quebec, Canada.

W. J. Wagner, A.M. (Illinois). Asst., Univ. of Illinois, Urbana, Ill.

Elizabeth V. Watts, A.M. (Chicago). Instr., Randolph-Macon Woman's Coll., Lynchburg, Va.

E. T. White, D. Paed. (Toronto). Math. master, Normal School, London, Canada.

D. E. Whitford, A.M. (Brown). Instr., Univ. of Rochester, Rochester, N. Y.

R. R. Wood, B.S. (Haverford). Instr., Whittier College, Whittier, Calif.
E. W. Woolard, Meteorologist, U. S. Weather Bureau, Washington, D. C.
A. L. Young, B.S. in M.E. (Iowa State). Instr., Iowa State Coll., Ames, Ia.

To Institutional Membership.

Crane Junior College, Chicago, Ill.
St. Xavier College for Women, Chicago, Ill.
Michigan State Normal College, Ypsilanti, Mich.
St. Benedict's College, St. Joseph, Minn.

The following have been appointed Associate Editors of the Monthly for the year 1922:

Julian L. Coolidge,
Edward L. Dodd,
Otto Dunkel,
Benjamin F. Finkel,
Walter B. Ford,
David C. Gillespie,
Cuthbert F. Gummer,
Leander M. Hoskins,
Raymond E. McClenon,
Francis D. Murnaghan,
David E. Smith.

Professor H. L. Rietz and the Secretary were appointed the representatives of the Association on the Council of the American Association for the Advancement of Science for 1922.

In order to facilitate the counting of ballots for the annual election, the Secretary was appointed a permanent member of the Committee of Tellers, this committee to consist of three members.

Professor Slaught reported that Chancellor A. B. Chace, of Brown University, had promised a contribution of $200.00 for 1922 toward the expenses of the Association. This was accepted with thanks and Professor Slaught was authorized to formulate a suitable reply to be sent to him and to be filed in duplicate with these minutes.

A revised report was made by the Association's Committee on Life-Membership, recommending that the life-membership fee be fixed at the exact value—at the attained age of the member—of the dues payable during his lifetime according to the McClintock 4% Male Table, using a deduction of 5% as the probable saving in collection expenses. The question was raised whether so complicated conditions should be made a part of the By-Laws, and a modification of the amendment proposed earlier, whereby the membership fee should be $105 - x$ dollars, was referred to the Association's committee for approval or suggestions.

A committee was appointed to consider the time and place for the summer meeting in 1922; also a committee to consider the general policy with reference to the Association library. Other matters of current business were transacted by the outgoing Board and the new Board for 1922 held an organization meeting after the election of new members by the Association on Friday.

Annual Business Meeting of the Association.

The Secretary-Treasurer announced the names of those elected to membership by the Board. He reported also the death of the following members:

PAUL ARNOLD, Professor of mathematics, University of Southern California (February 24, 1921);
J. E. CLARK, Professor of mathematics, Emeritus, Yale University (January 3, 1921);
N. F. DAVIS, Professor of mathematics, Emeritus, Brown University (May 17, 1921);
A. M. KENYON, Professor of mathematics, Purdue University (July 27, 1921);
H. R. PARK, Teacher, Junior College, Riverside, California (November 12, 1919);
ALEXANDER PELL, Bryn Mawr, Pa. (January 26, 1921);
ANNA I. YOUNG, Professor of mathematics, Agnes Scott College (September 30, 1920).

The election of officers for the year 1922 was conducted by mail and in person at this meeting, as provided in the By-Laws. The tellers (C. S. Atchison and A. D. Pitcher) reported the result of the balloting, 437 ballots having been cast, 2 of these for names not nominated on the official ballot:

For President: R. C. Archibald, 292 votes; J. L. Coolidge, 143 votes.

For Vice-Presidents: W. H. Bussey, 199 votes; R. D. Carmichael, 317 votes; B. F. Finkel, 240 votes; Paul Saurel, 89 votes.

For additional members of the Board of Trustees (to serve until January, 1925): Clara L. Bacon, 181 votes; L. P. Eisenhart, 253 votes; E. V. Huntington, 277 votes; W. D. Lambert, 190 votes; D. N. Lehmer, 232 votes; Helen A. Merrill, 214 votes; G. A. Miller, 269 votes; W. R. Ransom, 121 votes.

For member of the Board to serve until January, 1924, in the place of Professor Bennett (appointed Editor-in-Chief): Dunham Jackson, 187 votes; E. J. Wilczynski, 243 votes.

The following were accordingly declared elected:

President: R. C. ARCHIBALD, Brown University.

Vice-Presidents: R. D. CARMICHAEL, University of Illinois; B. F. FINKEL, Drury College.

Additional members of the Board of Trustees: L. P. EISENHART, Princeton University; E. V. HUNTINGTON, Harvard University; D. N. LEHMER, University of California; G. A. MILLER, University of Illinois.

For member of the Board to serve until January, 1924: E. J. WILCZYNSKI, University of Chicago.

The Secretary-Treasurer made his financial report for the year, giving an account of all the business transacted for the Association up to December 6th, 1921. The report of the Auditing Committee (C. H. YEATON, H. E. SLAUGHT, and Mrs. ANNA J. PELL, chairman) was then made. The financial report is printed in full below.

Report of the Secretary-Treasurer as Treasurer, Dec. 6, 1921.

Receipts.

Balance Dec. 2, 1920		$3,781.76
1920 indiv. dues	$ 131.80	
1920 instit. dues	32.50	
1921 indiv. dues	4,394.76	
1921 instit. dues	564.60	
1921 subscriptions	696.03	
Contributions to 1921 expenses	202.60	
Initiation fees	322.00	
Sale copies of Monthly	96.98	
Advertising	493.00	
Register	2.50	
Interest State Savgs. Bk.	74.91	
Interest Peoples Bk.	49.15	
Interest Liberty Bonds	45.00	
Total 1921 receipts		$7,105.83
Total assets to the end of 1921 business		$10,887.59
Total expenditures		$7,445.71
Balance to the end of 1921 business		$3,441.88
Received on 1922 and later business		572.10
Book balance Dec. 6, 1921		$4,013.98

Expenditures.

Publisher's bills (Nov. '20–Sept. '21)		$4,688.94
President's office		60.75
Manager's office		25.24
Editor-in-Chief's office		1,198.46
Other editors' postage		7.76
Secretary-Treasurer's office:		
Postage	$193.52	
Bond	5.00	
Office supplies	34.92	
Express, tel., etc.	12.23	
Clerical work	308.00	
Part expenses Register	27.00	
Printing	546.83	
Chicago annual meeting	55.73	
Wellesley meeting	73.84	
Paid copies of Monthly	6.25	
Paid to sections from initiation fees	51.24	
		$1,314.56
Annals subvention		150.00
Total expenditures		$7,445.71
Cash on hand		13.05
Checking account		83.66
State Savgs. Bk. Co. account		1,810.24
Peoples Bkg. Co. account		1,107.03
Liberty Bond		500.00
Victory Bond		500.00
Bank balance Dec. 6, 1921		$4,013.98

When the accounts were closed on December 6, 1921, in order to furnish the auditing committee a complete record, there remained on the total business for the year 1921, the following items:

Bills Receivable.

1921 dues unpaid	$150.00
Advertising	100.00
Interest Liberty Bonds	10.00
	$260.00

Bills Payable.

(Either paid in December or estimated.)

Publisher's bills (Oct.–Dec.)	$1,500.00
Manager's office	30.00
Editor-in-chief's office	80.00
Other editors' postage	20.00
Secretary-treasurer's office	190.00
Annals subvention	100.00
Init. fees due to sections	100.00
Printing annual ballots, programs, etc.	150.00
	$2,170.00

If to the balance on 1921 business shown in this report, $3,441.88, there be added the amount of bills receivable, $260, and there be subtracted the estimated amount of bills payable, \$2,170, there results an estimated final balance on 1921 business of approximately \$1,530. The corresponding estimated final balance one year ago on 1920 business was \$1,360. The apparently better showing of \$170 in 1921 over 1920 is more than accounted for by the "special contributions" of \$202.60 in 1921; thus the Association *has just been able to match expenditures with income.* It is only by the most careful attention to the affairs of the Association, by soliciting additional advertising, by repeated appeals to part of our members to pay their dues, and by similar time-consuming efforts, that the officers are able to make this showing. A greater margin of safety must be attained and this can be done through the coöperation of all members, especially in seeking steadily for new members.

W. D. CAIRNS, *Secretary-Treasurer.*

SERRET'S ANALOGUE OF DESARGUES'S THEOREM.

By H. S. WHITE, Vassar College.

In the course of a discussion of the eight points that lie on three quadric surfaces but not on any gauche cubic curve, P. Serret announced (*Géométrie de Direction,* 1869, page 325) an extension of Desargues's theorem concerning a quadrangle inscribed in a conic. This extension states an involutional property of a complete hexagon whose six vertices lie on a gauche cubic. Such a hexagon has 20 plane faces, or 10 pairs of opposite plane faces, each pair containing all six vertices. Any chord of the gauche cubic meets the cubic in points P and Q, and is cut by the 20 planes in 10 pairs of points. The theorem is: *The 10 pairs of points lie in a quadric involution, and* P, Q *form an 11th pair in the same involution.*

The proof, being inclusive of other things, is unnecessarily long for this particular purpose, and it is worth while to notice that the ordinary synthetic proof of the theorem on conics can be applied with very little change to this extension. Indeed, the only change required is to use pencils of planes through two chords of a cubic curve in place of the flat pencils of lines through two points of a conic.

Let P and Q denote two points on a gauche cubic curve. Six other points shall be called A, A_1, B, B_1, C, C_1. Consider the curve as generated by three projective pencils of planes, two of which have for axes the chords AA_1 and BB_1 respectively. Four pairs of corresponding planes will be

$$\begin{Bmatrix} AA_1P \\ BB_1P \end{Bmatrix}, \begin{Bmatrix} AA_1Q \\ BB_1Q \end{Bmatrix}, \begin{Bmatrix} AA_1C \\ BB_1C \end{Bmatrix}, \begin{Bmatrix} AA_1C_1 \\ BB_1C_1 \end{Bmatrix}.$$

Permute the four planes through BB_1 from the order $PQCC_1$ to the projective order QPC_1C. Then take sections of the two sets of planes by the chord PQ.